

Learning difficulties in mathematics – what are their nature and origin, and what can we do to counteract them?

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Around the world, more and more students are exposed to mathematics as a compulsory subject without having chosen it themselves, also at rather advanced levels. Large numbers of students who work hard to come to grips with the subject experience severe difficulties at learning mathematics and get no real success out of their efforts. Many of them do well in other subjects so their learning difficulties are not of a general nature, but specific to mathematics. If we want to help such students overcome their learning difficulties, or at least to reduce them, we have to be able to identify these difficulties, to understand their nature and their origin and to devise effective ways to counteract them.

Fifty years of research in mathematics education has given us valuable insights into the *learning* of mathematics. We now know quite a lot about how it takes place (or does not take place), and even if there is, of course, large variation in the learning of mathematics across individuals, societies and cultures, there are also significant commonalities, sometimes even universalities. As Celia Hoyles has pointed out, this is in contrast to what we find in the *teaching* of mathematics, which is much more dependent on national and local boundary conditions, socio-economic and cultural factors, history, philosophy, traditions and the structure and organization of the educational system. This is one of the reasons why, to the surprise and dismay of politicians and administrators, it is seldom possible to transfer successful teaching from one setting to another without substantial changes. Another reason is that it is difficult to capture and come to grips with those components of successful teaching that are responsible for successes observed.

Where to look for the most significant reasons why, everywhere in the world, mathematics is a difficult subject to learn? There are at least four sources to examine further:

- The nature of mathematics as a discipline
- The nature of human cognition
- The arenas and life spheres in which people can gain experiences that are conducive to the learning of mathematics
- The opportunities to learn that students are offered within the educational context in which they are embedded.

In the talk I shall present an outline of an analysis of these four sources, whilst paying particular attention to the first one.

Of course, the most important thing is to do something, if we can, about the learning difficulties that students experience. We know that lack of success in the endeavour to learn mathematics gives rise to severe problems for students, not only in terms of their future educational and occupational perspectives but also in terms of their personal intellectual and emotional development and self-esteem. So, this is an worthy task to undertake.

In the remainder of the talk I shall present and discuss a development and research programme that I am directing (together with my colleague Uffe Jankvist), which attempts at counteracting mathematics

specific learning difficulties identified with upper secondary school (high school) students (grades 10-12) in Denmark. The programme, which began in 2012, is a part-time in-service further education offer to practising teachers, enabling them to assist upper secondary school students with mathematics specific learning difficulties to overcome these as much as possible. The programme is closely tied up with teachers' everyday practice and runs over three semesters, each of which with a particular theme. The themes are: concepts and concept formation in mathematics, reasoning and proof in mathematics, and mathematical models and mathematical modelling. For each semester teachers do a theme specific project in small groups working with actual students and study a fair amount of research publications that are meant to underpin their project work. After having completed three projects the participant teachers are assessed at a final exam and, if they pass, receive a diploma issued by my university.

In the talk, details from this programme will be presented with a particular focus on the kinds of stumbling blocks students encounter and participating teachers interventions towards remedying these. The results of the programme up till now will be presented and discussed. Also, the programme has given rise to a number of research and development publications, and more are in the pipeline.